Discount Rate for Workout Recoveries: An Empirical Study*

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Abstract

Banks must measure the loss arising from counterparty default in order to achieve Advanced Internal Ratings-Based (IRB) compliance under the proposed Basel II minimum regulatory capital framework. The discount rate to be used on cash received post-default is not agreed upon amongst practitioners and banking supervisors. We review alternative extant proposals and develop a framework for choosing an appropriate discount rate contingent upon the risk of the recovery cash flow. Empirical results are presented by using a comprehensive database of trading prices and workout recoveries of both distressed bonds and loans. We find that discount rates vary significantly by initial issuer ratings, whether or not the industry is in stress at the time of default, relative seniority to other debt and instrument type. The conclusions are found to be robust to potentially confounding determinants of discount rate.

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* The views and opinions expressed here are those of the authors, not necessarily the views and opinions of the authors' employers, Standard & Poor's, Federal Reserve Bank of Richmond, the Federal Reserve Board of Governors, or the Federal Reserve System.

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I- Introduction

Implementation of the advanced internal ratings-based (IRB) approach under the Basel II minimum regulatory capital framework has been a major area of effort for many financial institutions. Among the issues debated in the industry is the appropriate discount rate to be used to estimate the loss given default (LGD). Which discount rate to use on workout recoveries post-default, in order to arrive at the "economic" value of the loan at the time of default, is a question that is the subject of considerable disagreement amongst practitioners and banking supervisors.[1]

Theoretically, the discount rate used in the estimation of LGD should be commensurate with the (opportunity) costs of holding the defaulted asset over the workout period, including an appropriate risk premium required by the asset holders.[2] This is also the typical interpretation adopted by the practitioners and Basel. For example, the Basel document states that: "when recovery streams are uncertain and involve risks that cannot be diversified away, net present value calculations must reflect the time value of money and a risk premium appropriate to the undiversifiable risk."[3]

Both practitioners and academicians approach the issue from a mostly theoretical standpoint and several quite different arguments have been made. The discount rate used ranges from funding rate, distress loan rate, contract rate, current comparable market rates, opportunity cost of funds, cost of capital, an accounting convention of the discount rate, and the current risk-free rate.[4]

Moreover, it is rarely the case that practitioners attempt to assign differentiated discount rates to different instruments according to the inherent degree of riskiness.[5] Without an appropriate risk adjustment, one could have consistently over- (under-) estimated the LGD of instruments with low (high) recovery risk. For an advanced IRB institution, it can translate into assigning disproportionately high (low) regulatory capital to instruments with low (high) recovery risk.

In this study, we attempt to shed some light on this issue by conducting an empirical study utilizing Standard & Poor (S&P)'s LossStats® Database, which is one of the largest databases commercially available capturing both workout recoveries and

[1] LGD can be estimated either by observing the market price of the defaulted instrument immediately after default or by computing the present value of future workout recoveries. Since the trading of most defaulted bank loans in the secondary market is illiquid, financial institutions typically have to rely on the latter approach in assessing their LGD. In the latter approach, LGD can be defined as the sum of the workout recoveries discounted at the *appropriate* discount rate divided by the total exposure at default.

[2] In the context of this study, the discount rate is equivalent to the expected or required rate of return of the asset holders (or investors of defaulted instruments). These terms are used interchangeably throughout the paper.

[3] See "Guidance on Paragraph 468 of the Framework Document," Basel Committee on Banking Supervision July 2005.

[4] For example, see Maclachlan (2004) and Davis (2004).

[5] In this paper, we are focusing on LGD risk rather than probability of default (PD) risk. The former measures recovery uncertainty after default has occurred.

defaulted loan and bond prices. We examine the relation between the market prices and the actual workout recoveries of 1,128 defaulted loans and bonds from 1987 to 2005 issued by 446 defaulted obligors. We develop an estimation methodology and estimate the discount rates, which are most likely used at the time of pricing by the market, reflecting LGD uncertainty for the defaulted loans and bonds.

We solve for the discount rate by interpreting it as the expected return of an investment in the defaulted instrument right after default has occurred. Such an investor pays for the instrument at the market price observed immediately after default in return for the future cash flows that could be realized during the workout process. In order to price the instrument appropriately, the investor needs to assess the expected future recoveries and the required risk premium, which is commensurate with the LGD uncertainty. In this study, we want to solve for the latter by estimating the former. Expected future recoveries are, however, not observable. Nevertheless, if the market is rational, the expected recovery should be identical to the average of the actual realized recoveries for a *reasonably* large and homogenous sample of default instruments in terms of their LGD uncertainty. Under this assumption, for each homogeneous segment of the dataset, we solve for the discount rate that equates the discounted average realized recoveries to the average market price.

We segment the data according to whether the instrument is secured or not, whether the instrument is rated speculative grade or not, the industry sector it belongs to, whether the instrument defaulted during the market-wide and industry-specific stress periods or not, the existence of *debt above* (DA) and *debt cushion* (DC) in the capital structure of the obligor, and its instrument types. Through the empirical analysis, we want to identify which are the important determinants of the estimated discount rates. We also provide the theoretical explanations together with the related business intuitions for the empirical results.

We attempt to validate our results by also conducting the analysis at the sub-segment level. We ask ourselves question like: "If we only consider Senior Unsecured Bonds, is the discount rate of an instrument defaulted during an industry-specific stress period significantly different from that defaulted outside such period?" The sub-segment level analysis allows us to control for the potentially confounding determinants of discount rate. Furthermore, we confirm the robustness of our findings by conducting a multiple regression analysis, which caters for the interaction among all the significant determinants of discount rates.

For each segment, as data permit, we compute the point estimate of the discount rate together with the corresponding confidence intervals around it. These results can be used by practitioners in estimating LGD based on their internal workout recovery data. For a quick reference, the most robust determinants together with the point estimates of discount rates are presented in Table 1. More detailed information is provided in the subsequent sections regarding the definition of segments, the estimation methodology and the interpretation of the results reported in Table 1.

Table 1: Point Estimate of Discount Rate

	Point Estimate of Discount Rate (%)
Overall	**14.0**
Investment Grade	22.8
Non-Investment Grade	6.4
Industry in stress	**21.5**
Industry not in stress	**8.1**
No Debt Above, Some Debt Cushion	21.2
No Debt Above and No Debt Cushion	0.9
No Debt Cushion, Some Debt Above	8.6
Some Debt Above and Some Debt Cushion	29.3
Bank Debt	**13.3**
Senior Secured Bonds	**11.0**
Senior Unsecured Bonds	**27.5**
Senior Subordinated Bonds	**3.8**
Subordinated Bonds	**8.9**

The following further summarizes the results:

(1) Our analysis suggests that whether the instrument is investment grade (IG) or non-investment grade (NIG), whether it defaults during an industry-specific stress period, the existence of DA and/or DC, and instrument type are important determinants of recovery risk and thus discount rate. Their statistical significances are confirmed in the subsequent sub-segment level analysis and the multiple regression analysis on the internal rate of returns of defaulted instruments.

(2) Defaulted debts of an originally highly-rated obligor tend to be more risky and command a higher risk premium. The implicit discount rate of an instrument defaulted from IG can be as much as four times higher than that of an instrument defaulted from NIG. Moreover, the *fallen angles* (i.e. those are IG in the earliest rating, while NIG one year before default) appear to have the highest uncertainty around their expected recoveries and also the highest expected return.

(3) The estimated discount rate of those debts defaulted during industry-specific stress periods is more than double of those defaulted outside those periods. Moreover, our results suggest the industry-specific stress condition is more important than the market-wide stress condition in determining the appropriate discount rates.

(4) The sub-segment results and the regression analysis do not support the Secured vs. Unsecured differentiation. It is however likely to be due to data constraints.[6] The impact of industry characteristics (e.g. technology vs. non-technology) and market-wide stress condition are found to be inconclusive.

(5) During market-wide stress periods when default rates are high, not only expected recoveries can be lower but also the discount rate is larger, which results in an even larger economic LGD. This is the idea that, during the stress periods, not only the expected recovery goes down but also the recovery uncertainly increases. As a result, the correlation between default rates and economic LGD would be more pronounced when this increase in the discount rate is accounted for during market downturns.

(6) Realized recoveries have a very large dispersion. As a result, the mean LGD calculated from a small sample of workout recoveries is subject to a high level of error. When this is the case, market price appears to be a better alternative to the discounted workout recoveries in determining the economic LGD.

(7) In general, investors demand a higher rate of return (i.e. discount rate of future cash flows) on defaulted instruments with a higher LGD risk, providing empirical support for an appropriate risk-return tradeoff. The sum of square errors of the realized recoveries is found to be positively related to the expected return on the defaulted instruments.

In the rest of this paper, we first introduce the dataset in Section II and the proposed segmentations together with their justifications in Section III. In Section IV, we explain the estimation methodology in detail. We report and interpret the empirical results in Section V where relevant business intuitions are also discussed. In Section VI, we validate our results by conducting sub-segment level analysis. We further examine the robustness of our conclusions by performing multiple regression analysis in Section VII. Finally, we conclude with a few remarks in Section VIII.

[6]The data does not allow us to distinguish the level of security, only the security type. That is, we could have two loans listed as secured, but the first one may be 100% secured, whereas the second one may only be 5% secured. It is very possible that this limitation weakens the significance of the Secured vs. Unsecured differentiation.

II-Data

To estimate the discount rates, we extract the market prices, workout recoveries and default rates data from the CreditPro® and the LossStats® Database, which are licensed by Standard & Poor's Risk Solutions.

LGD data

LossStats Database incorporates a comprehensive set of commercially assembled credit loss information on defaulted loans and bonds.[7] Public and private companies, both rated and non-rated, that have U.S.-issued bank loans and/or bonds of more than fifty million dollars are analyzed and included in the database. Financial, real estate, and insurance companies are excluded. The companies must have fully completed their restructuring, and all recovery information must be available in order to be included in the LossStats Database. The database contains recovery information from 1987 through the second quarter of 2005. We choose the LossStats Database as a reliable source of data due to its unique feature in that it contains both the *30-day distressed debt trading prices* and the *ultimate recovery* values of the defaulted instruments, both of which are required in this study. The 30-day distressed debt trading prices are simply the average trading prices of the defaulted instruments 30 days after the given default events. In contrast, ultimate recovery is the value a pre-petition creditor would have received had they held onto their position from the point of default through the emergence date from the restructuring event.[8]

A total of 1,128 defaulted instruments with both ultimate recovery values and 30-day distressed trading prices are included in the analysis.[9] These instruments are from 446 separate obligor default events from 1987 to 2005, and from a variety of industries. Table 2 reports the breakdown of the dataset according to security, S&P's rating and instrument type.

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[7] The October 2005 release v1.5 of the LossStats Database is utilized for this paper.

[8] Pre-petition creditors are creditors that were in place prior to filing a petition for bankruptcy.

[9] A total of 28 defaulted instruments with internal rate of returns of more than 800% are removed from the original dataset to ensure the results are not affected by these outliers. Moreover, among the remaining 1,128 defaulted instruments, some of them potentially represent duplicated observations referring to an identical instrument issued by the same obligor. To check for the robustness of our results, we repeat the analysis but with all the potentially duplicated observations excluded. The resulting discount rates are not materially different from those reported in this paper.

Table 2: Composition of LGD Data

Security	Secured	Unsecured				
	317	811				
S&P's Rating[10]	Investment grade	Non-investment grade	Others			
	88	398	642			
Type	Bank debt	Senior secured bonds	Senior unsecured bonds	Senior subordinated bonds	Subordinated bonds	Junior subordinated bonds
	222	102	395	237	161	11

Although the LossStats Database contains recovery information on bankruptcies, distressed exchanges, and other reorganization events, we only take into account companies, which had gone through a formal bankruptcy. This helps ensure the consistency between the 30-day distressed debt trading prices and the ultimate recovery values.[11]

Default rate data

We use the commercially available CreditPro® Database of ratings performance statistics to compile the aggregate default rates by industry. This information is then used in segmenting the data in the subsequent analysis. CreditPro, a subscription database of S&P's corporate rating histories globally, allows us to produce the time series of default rates for Global Industry Classification Standard (GICS) industry code classifications for S&P's-rated companies located in the U.S.[12]

III-Segmentation

We consider several factors that may influence the appropriate discount rate to apply to post-default recoveries when estimating economic LGD. In this section we describe the criteria we use to segment our dataset.

Secured/Unsecured describes whether the instrument is secured by collateral or not.

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[10] The classification is based on the earliest S&P's ratings of the instruments. Instruments from obligors that were not rated by S&P are classified as "Others".

[11] Since a distressed exchange (exchanging an instrument for an instrument of lesser value), can be considered a default event, and the completion of the exchange is considered a recovery event, they would often occur on or close to the same date. Therefore if we were to include these cases and use trading prices of the defaulted instrument 30 days after the default event, all information regarding the exchange would be known and there would be no uncertainty as to the recovery event.

[12] The Global Industry Classification Standard (GICS®) is an international industry classification system. It was jointly developed by S&P's and Morgan Stanley Capital International (MSCI) in 1999, as a response to the global financial community's need for one complete, consistent set of global sector and industry definitions. The highest industry sectors of GICS are found to be broader than those of the corresponding SIC codes.

S&P's Ratings: We segment our data based on whether the obligor is rated 'BBB-' and above (investment grade (IG)) or 'BB+' and below (non-investment grade (NIG)) by S&P. We consider segmenting according to the earliest S&P rating and the S&P rating one year prior to default respectively. S&P's obligor ratings indicate the amount of certainty that creditors have in their ability to receive all claims in full, meaning that companies with higher credit ratings would be less likely to default. Claims on issuers with higher ratings may lack provisions that would provide creditors with higher recovery rates; however, highly-rated companies may also have more reliable business plans that would lead to a higher price on exit from default.

GICS® Industry Codes describe the industry grouping to which each obligor belongs. Industry may affect expected recoveries because investors expect companies in certain industries to have a greater ability to deliver on a post-default business plan or to produce superior recoveries to other industries. **GICS Groupings** are broad industry groupings that combine various 2-digit GICS codes. We aggregate 2-digit GICS in order to produce industry groups of sufficient sample size for statistical analysis. In the subsequent analysis, we focus our attention on the difference between the technology and non-technology sector.

Default in Market-Wide Stress Period describes whether the obligor defaults during a market-wide stress period or not. In this study, market-wide stress periods are defined as those years (1986, 1990-1992 and 1999-2003) where the speculative grade default rates are greater than its 25-year average of 4.7%. Market-wide stress periods may be filled with more uncertainty and pessimism, which would drive down the post-default trading price as well as increase the expected discount rates for post-default recoveries. It is also plausible that due to the correlation between probability of default (PD) and LGD, obligors' asset and collateral values are depressed during periods of high default rates.

Default in Industry-Specific Stress Period describes whether the obligor defaults during those periods where the obligor's industry (based on the GICS code) experiences a speculative-grade default rate of more than 5% (which is the long-run average default rate for all speculative-grade obligors in the U.S.). In combining with the previous consideration of market-wide stress period, we can therefore examine whether it is industry-specific or market-wide downturn, which drives the potentially lower expectations of future recoveries.

Debt Above (DA) and Debt Cushion (DC) describe whether there is debt that is superior to or subordinated to each bond and bank loan for the given default event. DA is the sum of the amount of debt instruments which are contractually superior to the instrument that is being analyzed divided by total amount of debt. This is in contrast to DC, which is the sum of the amount of debt instruments which are contractually inferior to the instrument that is being analyzed divided by total amount of debt. Due to the variability of the capital structure of defaulted obligor, by defining according to its DA and DC, instrument can be more readily compared than by classifying according to the name of the instrument alone (e.g. Senior Secured Bonds). In this study, we segment our sample into: (1) those with no DA and some DC; (2) those with no DA and no DC (3) those with no DC and some DA; and (4) those with some DA and some DC.

Instrument Type groupings are based on the legal description of the instrument. Instrument type is frequently used in practice in classifying instruments for LGD assessments. Similar to DA and DC, instrument type provides information about the seniority of the creditor within the list of claimants. However, the same instrument type can represent very different instruments when we compare obligors with very different debt structure. For example, a subordinated bond issued by an obligor having only a single class of debt may have a much lower recovery risk than a subordinated bond issued by another obligor which also issues senior bonds. Classifying by DA and DC is considered to be a more appropriate way to control for any differences in debt structure across obligors. In this study, we consider the instrument types of: (1) Bank Debt (2) Senior Secured Bonds, (3) Senior Unsecured Bonds, (4) Senior Subordinated Bonds, (5) Subordinated Bonds, and (6) Junior Subordinated Bonds.

IV-Estimation of Discount Rate

We estimate the discount rate for each segment by modeling it as the expected rate of return on the investments in defaulted instruments belonging to that segment. We assume all instruments within a particular segment are identical in terms of their LGD risk and thus share the same expected rate of return. The fact that the realized recovery turns out to be different from the expected recovery is solely because of LGD uncertainty during the recovery process.

Specifically, consider an instrument *i* defaults at time t_i^D, while a single recovery cash flow R_i is realized at time t_i^R. We observe the market price (P_i) of this defaulted instrument at 30 days after it defaults. If we use *d* to denote the expected return (i.e. the discount rate) on investing in this defaulted instrument, the price of the instrument is the expected recovery discounted at *d*, which is the parameter we want to estimate in this study.

$$P_i = \frac{E[R_i]}{(1+d)^{t_i^R - t_i^D - 30}} \tag{1}$$

Expected recovery, $E[R_i]$, is however not observable. We can only observe the realized recovery, R_i. Because of LGD uncertainty, we do not anticipate the expected and realized recovery to be identical unless by coincidence. However, if market is rational, the difference between the expected recovery and the *average* realized recovery should be small for a sufficiently large enough group of instruments that are homogeneous with respect to LGD risk. We can formalize this idea as below. Let ε_i represent the difference between the expected and realized recovery, being normalized by the price of the instrument, P_i.

$$\varepsilon_i = \frac{R_i - E[R_i]}{P_i} \text{ or equivalently} \tag{2}$$

$$\varepsilon_i = \frac{R_i - P_i \times (1+d)^{t_i^R - t_i^D - 30}}{P_i} \tag{3}$$

It can be interpreted as the *unexpected* return of the defaulted instrument *i*. It is, therefore, a *unit-free* measure of recovery uncertainty. However, before we can solve for the discount rate, we also need to model for the fact that recovery uncertainty can be a function of the time-to-recovery. As suggested by Miu and Ozdemir (2005), recovery uncertainty is likely to be an increasing function of the time-to-recovery since more and more information on LGD is revealed over time (i.e. less recovery uncertainty) as we approach the time when recovery is finally realized. In this study, we assume the standard deviation of ε_i is proportional to the square root of time-to-recovery (i.e. $t_i^R - t_i^D - 30\text{ days}$). This assumption is consistent with an economy where information is revealed uniformly and independently through time.[13] With this assumption and if the market is rational, $\varepsilon_i \big/ \sqrt{t_i^R - t_i^D - 30\text{ days}}$ should be small on average across a homogenous group of defaulted instruments. The *most-likely* discount rate ($\hat{d}$) can therefore be obtained by minimizing the sum of the square of $\varepsilon_i \big/ \sqrt{t_i^R - t_i^D - 30\text{ days}}$ of all the defaulted instruments belonging to a segment.[14] It serves as our point estimate of the discount rate for that particular segment. Formally,

$$\hat{d} = \arg\min \sum_{i=1}^{N} \left(\frac{\varepsilon_i}{\sqrt{t_i^R - t_i^D - 30\text{ days}}} \right)^2 \tag{4a}$$

where *N* is the number of defaulted instruments in that segment. A by-product of the above algorithm is the minimum sum of the squares of ε_i (SSE) normalized by the square root of the time-to-recovery. It can be interpreted as a standardized measure of the recovery uncertainty (i.e. LGD risk) of that specific segment.

The discount rate estimate obtained by solving equation (4a) is in fact the maximum likelihood estimate. That is,

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[13] Given the fact that the time-to-recovery of the instruments in our dataset ranges from a few days to more than a couple of years, the results can be significantly biased if we ignore the relation between LGD uncertainty and time-to-recovery. If we had assumed no time adjustment, the discount rates would have been slightly different from those presented here. However, the conclusions drawn in this study are robust whether we adjust for time or not.

[14] This sum of the squares of errors (SSE) of recovery returns can be used to proxy for LGD risk for each segment. An appropriate risk-return tradeoff suggests the discount rate (i.e. expected return) should be high when SSE (i.e. LGD uncertainty) is high.

$$\hat{d} = \arg\max L = \arg\max \sum_{i=1}^{N} \log\left[\phi\left(\frac{\varepsilon_i}{\sqrt{t_i^R - t_i^D - 30 \text{ days}}}\right)\right] \tag{4b}$$

where L is the log-likelihood function and $\phi(\bullet)$ is the probability density of the standard normal distribution. We can also estimate the asymptotic standard deviation $\sigma_{\hat{d}}$ of $\hat{d}$ by evaluating the second derivative of L with respect to d at the point estimate $\hat{d}$.

$$\sigma_{\hat{d}} = \left(-\frac{d^2 L}{dd^2}\right)_{d=\hat{d}}^{-0.5} \tag{5}$$

We can therefore establish confidence interval around our point estimate of discount rate. For example, the confidence interval at the 90% confidence level is equal to:[15]

$$\hat{d} \pm 1.6449 \times \sigma_{\hat{d}} \tag{6}$$

The above methodology can be readily extended to include default instruments with multiple recoveries. Suppose there are a total of m_i recovery cash flows ($R_{i,1}$, $R_{i,2}$, …, $R_{i,j}$, …, R_{i,m_i}) which are realized at times $t_{i,1}^R$, $t_{i,2}^R$, …, $t_{i,j}^R$, … and t_{i,m_i}^R respectively for defaulted instrument i. The percentage difference between expected and realized recovery cash flow j is therefore equal to:[16]

$$\varepsilon_{i,j} = \frac{R_{i,j} - p_{i,j} \times (1+d)^{t_{i,j}^R - t_i^D - 30}}{p_{i,j}} \tag{7}$$

where $P_i = \sum_{j=1}^{m_i} p_{i,j}$. The most-likely discount rate (i.e. our point estimate of the discount rate) can therefore be solved by:

$$\hat{d} = \arg\min \sum_{i=1}^{N} \sum_{j=1}^{m_i} \left(\frac{\varepsilon_{i,j}}{\sqrt{t_{i,j}^R - t_i^D - 30 \text{ days}}}\right)^2 \tag{8}$$

where the *outer* summation is across all N defaulted instruments of the segment. The corresponding asymptotic standard deviation and confidence interval of the point estimate can also be computed in a similar fashion as outlined above.

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[15] Note that (1-2×(1-Φ(1.6449))) = 90%, where Φ is the cumulative standard normal distribution function.

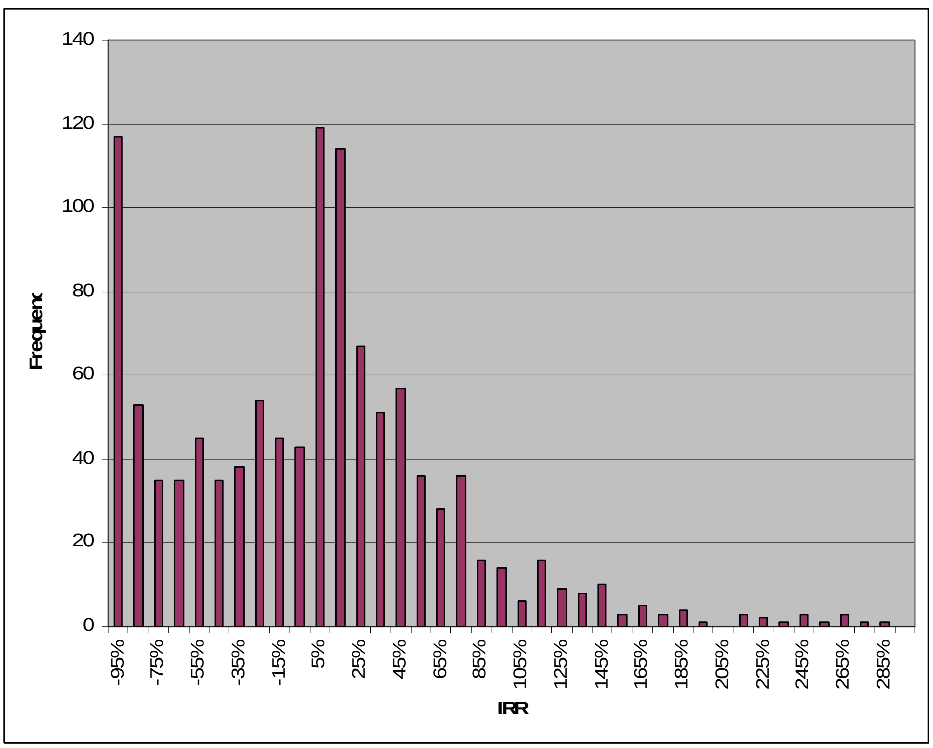
[16] $p_{i,j}$ are estimated in an iterative fashion such that they are proportional to the present values of the realized recovery cash flows $R_{i,j}$ discounted at the most-likely discount rate $\hat{d}$. In our sample, the majority of the default instruments have only single recovery.

V-Discount Rate

In this section, we report the discount rates estimated for each segment using the methodology outlined above. We also provide some interpretations on the results. It should be noted that the ultimate recoveries obtained from LossStats Database are *gross* recoveries rather than *net* recoveries. That is, any direct and indirect collection costs such as legal fees, overheads and other collection expenses typically encountered by a financial institution are not yet subtracted from the ultimate recoveries. For the purpose of measuring economic LGD, banks should apply the discount rate on their net recoveries rather than gross recoveries. By conducting our analysis using gross recoveries, we are therefore overestimating recoveries and thus the discount rate to be used for the above purpose. As a result, if a financial institution applies the discount rates estimated in this study to their net recoveries, the resulting LGD would be somewhat conservative.[17]

Before reporting the discount rates, we present the histogram (Figure 1) of the internal rate of returns of all individual instruments within our sample.

Figure 1: Histogram of Internal Rate of Returns of All Instruments



The negative rates presented in Figure 1 may appear to be counterintuitive, indication of "non-economic behaviour". This is not actually the case, as the negative return realized

[17] As an indication, using the full dataset, when a flat \$5 (i.e. 5% of notional), representing the collection cost is subtracted from the ultimate undiscounted recoveries, overall discount rate decreases from 14.0% to 12.1%. It is more appropriate to assign a cost of collection as a fixed % of the notional amount (i.e. \$1,000) rather than a fixed % of the actual recovery in order to accommodate zero recoveries.

may simply be due to recovery uncertainty. In practice, we expect to observe negative return on some investments. A negative return does not necessarily suggest that the investor over-estimated the value of the asset, but instead she may simply be unlucky. Investors price the defaulted instrument based on the expected value over a distribution of possible recovery cash flows. A negative rate of return may then be realized if the actual cash flows turn out to be on the "loss" side of the distribution

The internal rates of returns have a large variation as observed from Figure 1. It is a bimodal distribution with a *thick* right-hand tail. The distribution is truncated at -100% and at the same time very large positive returns with low frequency are observed, suggesting an "option-like" behaviour of the defaulted assets. If an investor purchases a defaulted instrument *randomly* immediately after default and holds it until it emerges, there is about a 43% chance that the investment will generate a negative return, while a 5% chance that she cannot recover anything from her investment. The potential profit however can also be very substantial.

Some banks, in their estimation of LGD, rely on both workout recoveries and market prices, and they consider them as substitutes to each other.[18] If the bank's primary practice is to sell its defaulted assets, LGD estimation needs to be based on market prices. However, if it is the bank's policy to work out its defaulted assets, LGD estimation needs to be based on discounted workout recoveries. Figure 1 suggests caution needs to be paid as realized recoveries have a very large dispersion; as a result, the mean LGD calculated from a small sample of workout recoveries is subject to a high level of error. Discounting workout recovery can therefore only serve as a good substitution of the respective market price when we have a sufficiently large and homogeneous group of facilities. The average of discounted workout recoveries, computed from individual or a small group of defaulted instruments is unlikely to be an accurate representation of the economic value of LGD.

Before we present the discount rates obtained by segmenting the dataset in different ways, we report in Table 3 the "Overall" discount rate estimated by using the whole dataset. The most-likely estimate is obtained by using the estimation methodology outlined in the previous section. We also report the corresponding asymptotic standard deviation and 90% confidence interval of the estimate. The results in Table 3 suggest that the discount rate is most-likely to be 14.0% if we assume all defaulted instruments are homogenous in its recovery risk. There is a 90% chance that the actual discount rate lies between 11.1% and 16.9%. These results can therefore serve as benchmarks when we consider the discount rates obtained for individual segments.

[18] Some banks only use market prices for those defaults that are still being worked-out and consider them as proxies for the ultimate recoveries.

Table 3: “Overall” Discount Rate in Percent

	Most Likely Estimate	Standard Deviation	90% Confidence Interval	
			Lower	Upper
Overall	**14.0**	1.8	11.1	16.9

Secured vs. Unsecured

The results in Table 4 suggest unsecured debts have higher discount rate than secured debts, which is as expected. Unsecured defaulted debts are riskier than secured ones, as creditors will only get what is left after secured are paid based upon the absolute priority rule, therefore resulting in greater uncertainty to the recovery, and thus investors require a larger risk premium.[19] Not only are the recoveries more uncertain (total risks) for unsecured debts, there is also likely to be a larger component of systematic risks. For example, the findings of Araten, Jacobs Jr. and Varshney (2004) suggest LGD of unsecured loans is more correlated with PD (and thus the general economy) than LGD of secured loans.

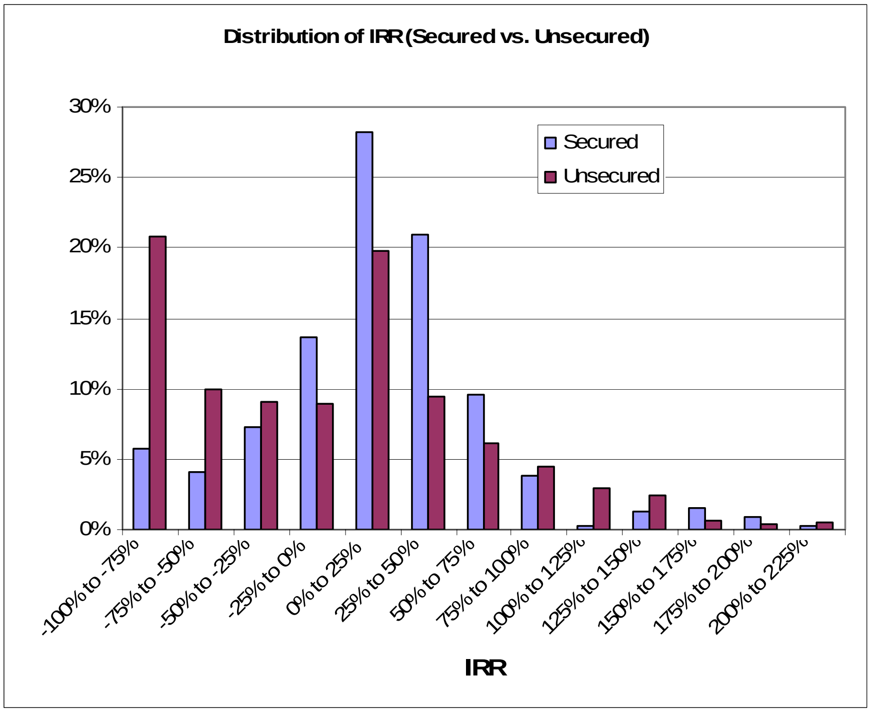
The idea of unsecured defaulted debts being riskier than secured debts is also supported by examining the realized annualized returns (i.e. internal rate of return) of individual instruments. The histogram reported in Figure 2 suggests it is more likely for the unsecured defaulted debts to generate significantly negative return (less than -25%) and significantly positive return (more than 75%) than the secured ones.

Table 4: Discount Rate in Percent: Secured vs. Unsecured Instruments

	Most Likely Estimate	Standard Deviation	90% Confidence Interval	
			Lower	Upper
Secured	**11.8**	4.8	3.9	19.7
Unsecured	**14.3**	1.9	11.2	17.4

[19] The absolute priority rule however may not necessarily always be observed in practice.

Figure 2: Histogram of Internal Rate of Returns: Secured vs. Unsecured



Despite of the strong intuition and the evidence from the earlier studies, the wide confidence intervals reported in Table 4 indicate that the results are not as strong as expected (refer to section VI for further discussion). This is most likely due to the fact that the data does not allow us to distinguish the level of security, only the security type. That is, we could have two loans listed as secured, but the first one may be 100% secured and where as the second one may be only 5% secured. It is likely that this weakens the differentiation between the secured and unsecured credits. The difference between the "highly" secured instruments and unsecured instruments might be significantly more pronounced if we were able to account for the exact degree of securitization.

Investment Grade (IG) vs. Non-Investment Grade (NIG)

From Table 5, the estimated discount rate of IG is much larger (almost 4 times) than that of NIG based on the earliest S&P rating. Non-overlapping confidence intervals indicate the power of discrimination. This finding is also found to be robust when we consider sub-segment results (refer to section VI for further discussion). It is actually not surprising that a higher level of recovery risk is implicit in an IG instrument rather than a NIG one. Since it is highly unanticipated that an IG debt defaults, the fact that it defaults increases the riskiness of the distress instruments. Moreover, creditors of an originally highly-rated obligor are much less concern about its LGD when the obligor is still a going concern since its default risk is perceived to be slim. They are likely to pay more attention in monitoring and mitigating the LGD risk of lowly-rated obligors rather than highly-rated ones. As a result, defaulted debts of an originally highly-rated obligor tend to be more risky and command a higher risk premium.

Table 5: Discount Rate in Percent: IG vs. NIG

	Most Likely Estimate	Standard Deviation	90% Confidence Interval	
			Lower	Upper
IG (earliest)	**22.8**	5.0	14.6	31.0
NIG (earliest)	**6.4**	3.8	0.2	12.7

We also examine if the rating history has any bearing on the discount rate by segmenting the data jointly according to whether they are investment grade in the earliest S&P ratings and in the S&P ratings assigned one year before the respective default events. The results are reported in Table 6.

Table 6: Discount Rate (in Percent) of Different Rating Histories. The corresponding 90% confidence intervals are presented in brackets.

	Most-Likely Discount Rate (%)	
	IG (earliest)	**NIG (earliest)**
IG (1 yr before)	14.9 (3.6 - 26.2)	N/A
NIG (1 yr before)	34.8 (21.1 – 48.6)	5.3 (0.0 – 12.1)

Note: Within our dataset, there are only three observations which are of NIG according to their earliest ratings and subsequently become IG one year before the respective default events. Results are therefore not reported for this case.

The *fallen angles* (i.e. those rated as IG in their earliest ratings while subsequently becoming NIG one year before their defaults) appear to have the highest uncertainty around their expected recoveries and thus require the highest discount rate. Fallen angels having higher recovery rates than original issued speculatively-rated debts may be explained by the nature of fallen angels. That is, they are obligors that used to have high credit quality before they defaulted. Some of those advantageous qualities may carry over into the default work out process. Additionally, when fallen angels default, market participants may be less likely to invest in their defaulted debt due to the extreme change in the fortunes of the obligor, thus demanding a larger discount rate than an otherwise identical obligor which has always been rated as NIG. It is interesting that fallen angels seem to have a combination of high discount rates and high recovery rates. These findings are in line with Vazza, Aurora and Schneck (2005) which note "that the post-default experience among investors is more favorable for fallen angels in the aggregate than for their counterparts. Even after adjusting for security type, fallen angels show more favorable recovery characteristics …"

Industry

Our intuition is that, those industries with exposures typically collateralized by tangible assets and real estate - of which values can readily be determined – would require a lower discount rate than those with exposures collateralized by assets of which values are more uncertain.

We study the industry effect by classifying the obligors in the dataset according to their GICS. We first compare technology based against non-technology based industries.[20] We expect defaulted instruments issued by the technology sector to have higher recovery uncertainty since they are collateralized by typically intangible assets. An originally secured instrument can become essentially "unsecured" when the collateral loses its perceived value. As an example the supposedly fully secured creditors involved in the *Winstar Communications, Inc.*'s 2001 bankruptcy, saw their positions practically being wiped out. As the overcapacity in the telecom industry leads to a large devaluation of the underlying collateral, the creditors saw their collateral values decline along with the rest of the telecom industry, essentially making the secured creditors into unsecured creditors. The discount rates reported in Table 7 substantiate the above hypothesis. The estimated discount rate of defaulted instruments of the technology industries is higher than that of the non-technology ones.

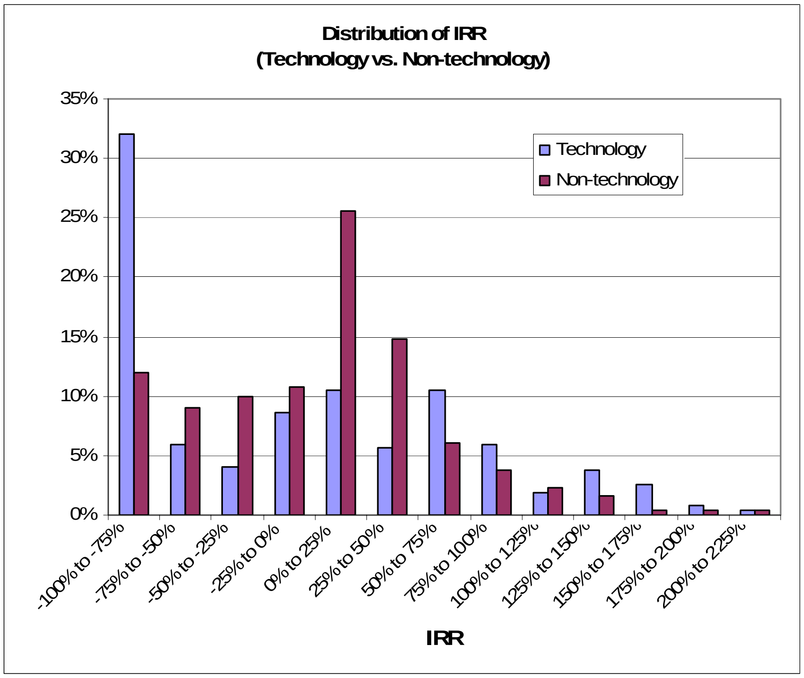
Table 7: Discount Rate in Percent: Technology vs. Non-Technology

	Most Likely Estimate	Standard Deviation	90% Confidence Interval	
			Lower	Upper
Technology	**24.4**	5.8	14.8	34.0
Non-Technology	**13.0**	1.9	9.8	16.2

The idea that defaulted instruments of technology based industries should be riskier than those of non-technology based is also supported by examining the internal rate of return of individual defaulted instruments. The histogram reported in Figure 3 suggests it is more likely for the former to generate significantly negative return (less than -75%) and significantly positive return (more than 50%) than the latter.

[20] In this study, Information Technology (GICS 40-45) and Telecommunication Services (GICS 50) are considered technology based industries, while others are non-technology based.

Figure 3: Histogram of Internal Rate of Returns: Technology vs. Non-Technology



The overlapping confidence intervals reported in Table 7 however indicate that the industry effect is not as strong as the other factor considered in this study (refer to section VI for further analysis of the related sub-segment results). We further examine the effect of collateral values by breaking down the non-technology sector into further sub-sectors, namely Energy & Utilities, Consumer Staples/Materials, but the results (not reported) are found to be inconclusive.

Default during Market-Wide Stress Period

In Table 8, we compare the estimated discount rate of those instruments defaulted during the market-wide stress periods with those defaulted outside the stress periods. In the last two columns of Table 8, we also decompose the estimated discount rates into their respective risk-free interest rates and risk premium components. To compute the implicit risk premium, we subtract from the respective discount rates the average yields of the 1-year US Treasuries, which serve as proxies for the risk-free rates over the respective periods.[21] As expected, given the lower risk-free rate during an economic downturn, the difference in risk premiums between the two states of the economy is even larger than that of the estimated discount rates.

[21] Monthly US Treasury yields from 1987 to 2005 are obtained from the US Federal Reserve Board.

Table 8: Discount Rate and Decomposition into Risk-Free Rate and Risk Premium (in Percent): Market-Wide Stress Condition

	Estimated Discount Rate				Decomposition	
	Most Likely Estimate	Standard Deviation	90% Confidence Interval		Ave. US Treasury Yield	Implicit Risk Premium
			Lower	Upper		
In Market-Wide Stress Period	**15.7**	4.2	8.8	22.6	4.4	**11.3**
Not in Market-Wide Stress Period	**13.6**	2.0	10.3	16.9	5.5	**8.1**

The finding that investors require a larger risk premium during a stress period is consistent with the fact that LGD risk is higher during the recessionary period. For example, the empirical studies of Araten, Jacobs Jr. and Varshney (2004) and Acharya, Bharath and Srinivasan (2003) document larger LGD variation during stress periods. The higher risk can be due to the fact that uncertainly around the values of the collaterals and the defaulted companies' assets may increase during a market-wide stress period.

This finding also has interesting implications with respect to the correlation between default rates and LGD. A positive correlation is already documented by, for example, Araten, Jacobs Jr. and Varshney (2004) and Miu and Ozdemir (2006), which is primarily due to the fact that during the period of high defaults, recoveries tend to be low as assets and collateral values are depressed. These studies use a single discount rate to discount the workout recoveries for all defaulted instruments and thus the discount rate effect studied here is not accounted for. Unlike these studies, we can attribute the positive relation between default rate and LGD to the variation of risk premium across the different states of the economy. In other words, our study shows that during recessionary periods when default rates are high, not only expected recoveries may be lower but also the discount rate is larger, which results in an even larger economic LGD. This is the idea that, during the recessionary periods, not only the expected recovery decreases but the recovery uncertainly also increases. As a result, the correlation between default rate and economic LGD would be more pronounced when the increase in discount rate documented in this study is accounted for during market downturns.

The higher discount rate during a stress period is also consistent with the theory of excess supply of defaulted debts during a market-wide stress period. Altman et al (2005) suggests that the increase in LGD during recessionary periods is mainly due to an increase in the supply of default instruments.

Judging from the overlapping confidence intervals reported in Table 8, the difference in discount rates between the market-wide stress and non-stress periods is, however, not strong. Specifically, the effect is much weaker than that between the industry-specific stress and non-stress periods as reported subsequently. It, therefore, suggests the industry-specific effect is a more important determinant of the expected return than the market-wide effect. This weaker market-wide effect is also documented in section VI when we conduct the sub-segment analysis.

Default during Industry-Specific Stress Period

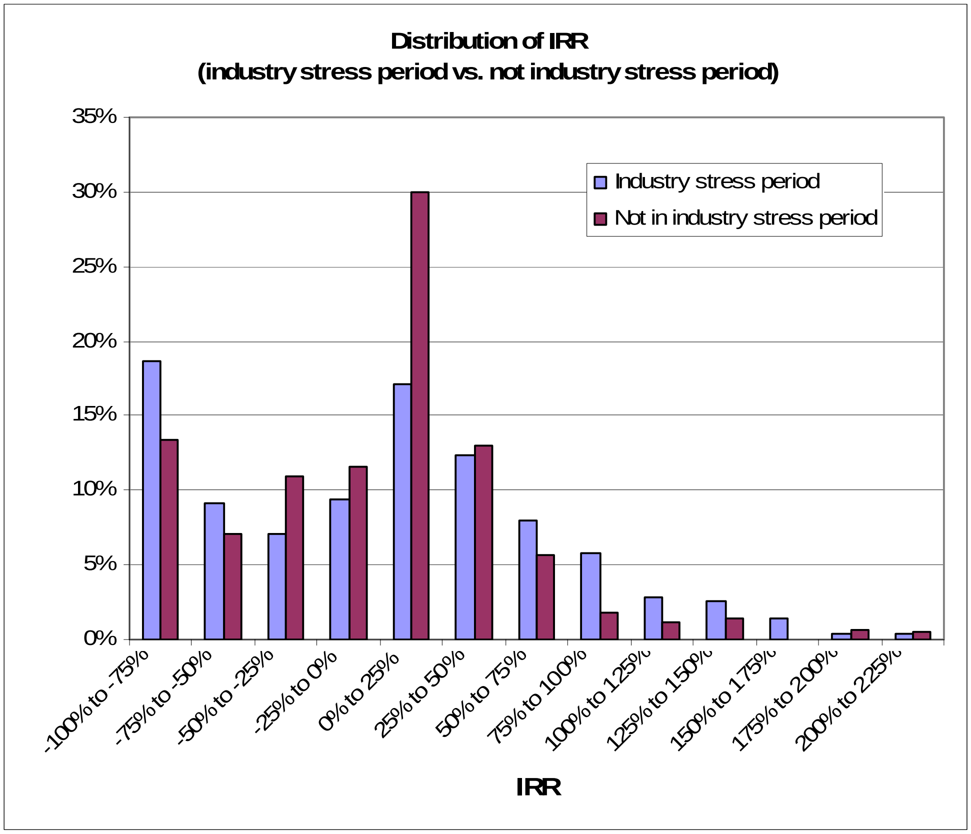
In Table 9, we compare the estimated discount rate of those instruments defaulted during the industry-specific stress periods with those defaulted outside the stress periods. Industry-specific stress periods are defined as those periods when the industry experiences a speculative-grade default rate of more than 5%. The estimated discount rate during the industry-specific stress period is found to be more than double of that when otherwise. Judging from the non-overlapping confidence intervals, the difference is highly statistically significant. The result is also found to be robust when we conduct the analysis at the sub-segment level in section VI.

Table 9: Discount Rate in Percent: Industry-Specific Stress Condition

	Most Likely Estimate	Standard Deviation	90% Confidence Interval	
			Lower	Upper
In Industry-Specific Stress Period	**21.5**	2.7	17.1	25.8
Not in Industry-Specific Stress Period	**8.1**	3.0	3.1	13.1

The idea that defaulted instruments during an industry stress period are riskier than those not in a stress period is also supported by examining the internal rate of return of individual defaulted instruments. The histogram reported in Figure 4 suggests it is more likely for the former to generate significantly negative return (less than -50%) and significantly positive return (more than 50%) than the latter.

Figure 4: Histogram of Internal Rate of Returns: Industry-Specific Stress Condition



Comparing the statistical significance of the results reported in Table 8 and 9, we can conclude industry-specific stress condition is more important than market-wide stress condition in governing the expected return on distress debt. This is consistent with the empirical findings of Acharya, Bharath and Srinivasan (2003) and the theoretical model proposed by Shleifer and Vishny (1992). Shleifer and Vishny suggest financial distress is more costly to borrowers if they default when their competitors in the same industry are experiencing cash flow problems. It therefore supports the concept that there are cross industry diversification effects in play, meaning that different industries have different LGD cycles.

Another plausible argument for industry undergoing stress situation leading to greater LGD uncertainty is that the uncertainly around the values of the collaterals increases during an industry-specific stress period, assuming that collaterals are mostly industry specific, for instance, fiber-optic cable for telecom sectors.

The observation that industry-specific stress condition is more important than market-wide stress condition has an interesting implication on the computation of capital requirement. Under Basel II Advanced IRB approach of regulatory capital calculation or under an economic capital framework, banks need to estimate LGD by discounting the workout recoveries at the appropriate discount rate. Capital is estimated at the extreme confidence interval, corresponding to extreme events, which is most likely to occur in stress periods. To fulfill this objective, the appropriate discount rate is arguably the one estimated under the market-wide stress period documented in this study. However, an argument can be made that the diversification effects across different industries should also be considered, since not all industries are likely to be under stress situation at the same time. Nevertheless, a capital event would be an extreme stress scenario causing a bank's capital to be wiped-out by credit losses. During such an extreme event, it is likely that many, maybe most, industries would be experiencing stress situation at the same time.

Since we are extracting the discount rate by using the market price of the defaulted instrument, our discount rate might also incorporate any short-term effects and risk measures specific to the secondary market of distressed instruments. Namely, as suggested by Altman et al (2005), the excess supply of defaulted instruments during a distressed period can by itself exert a downward pressure on the market price and thus translate into a further increase in the implicit discount rate. For those banks with a policy to always work out their defaulted assets rather than selling them in the secondary market, they would not need to concern themselves with this short term demand/supply issue in the secondary market and, thus, may need to adjust the discount rates reported in the study before using them in their calculations of the economic values of LGDs. A bank which does not sell distressed assets should not be expected to hold capital against the market risk that the prices of distressed instruments in the secondary market are depressed by oversupply during a market downturn.

DA and DC

From Table 10, the priority of the instruments on the balance sheet as measured by DA and DC appears to be an important driving factor in determining the required discount rate. Specifically, judging from the point estimates and their confidence intervals, those instruments with some DC (whether with or without DA) have statistically significantly higher required discount rate than those without DC (again, whether with or without DA). This effect of instrument structure on recovery is consistent with the findings of VandeCastle (2000). This result is also found to be robust when we conduct the sub-segment analysis in Section VI.

Table 10: Discount Rate in Percent: DA vs. DC

	Most Likely Estimate	Standard Deviation	90% Confidence Interval	
			Lower	Upper
No DA, Some DC	**21.2**	3.7	15.1	27.3
No DA and No DC	**0.9**	7.9	-12.1	13.8
No DC, Some DA	**8.6**	3.0	3.7	13.6
Some DA and Some DC	**29.3**	4.0	22.7	35.8

The amount of debt cushion and collateral securing a position not only has an effect on the expected recovery rate. The results in Table 10 suggests that knowing the debt structure of the given obligor and the position of the instrument being analyzed in this structure may also shed some light on the amount of uncertainty surrounding the expected recoveries.

If a given instrument has no DA or DC, this would mean that there are no other bonds and loans with either higher or lower priority on the balance sheet. All creditors therefore share equally in the underlying assets securing the positions; resulting in a fairly predictable recovery rate. As expected, we observe the lowest required discount rate for instruments with no DA and no DC.

Following the same argument, those instruments with some DA and some DC will have the most uncertainty surrounding the expected recovery, as there are both senior and junior positions who will be vying for a portion of the defaulted obligors' assets. As the creditors do not know how much they need to pay the subordinated creditors in order to move the debt restructure plan forward, or how much the senior creditors agree to pay them for the same reason. It is also consistent with the idea suggested by Acharya, Bharath and Srinivasan (2003) that more demanding the coordination effort among creditors in the situation of larger "debt dispersion" leads to higher bankruptcy and liquidation costs, thus translating into a higher required rate of return. As expected, this effect is found to be even stronger if we confine ourselves to only unsecured instruments. The estimated discount rate of those unsecured instruments with some DA and some DC is found to be equal to 32.1% (refer to the sub-segment results reported in Table 12 of section VI).

For those instruments belonging to the categories of "No DA, Some DC" and "No DC, Some DA", the estimated discount rates are in between those of the prior two categories discussed. From Table 10, instruments belonging to the category "No DC, Some DA" have a lower estimated discount rate between the two. Since these instruments are in the bottom of the list of absolute priority, creditors expect not only a relatively low level of recovery rate but also a low variation around the expected value.[23] It therefore results in a lower recovery uncertainty and in turn a lower required discount rate. Finally, those instruments belonging to the category of "No DA, Some DC" should have the highest expected recovery of all the four categories; however there may still be a large level of uncertainty to the ultimate recovery. This uncertainty therefore leads to a high required discount rate.

Instrument Type

From Table 11 and judging from the confidence intervals, we can conclude the estimated discount rate of senior unsecured instruments are statistically significantly higher than those of senior secured bonds (see also secured vs. unsecured discussed above), senior subordinated bonds and subordinated bonds. Senior subordinated bonds have the lowest

[23] Recovery cash flow always has a lower bound of zero.

point estimate of discount rate among all the types. The differences in discount rates should reflect the different degrees of uncertainty in the recovery process for each instrument type. For example, the relatively higher (lower) discount rate for senior unsecured bonds (subordinated bonds) suggests a higher (lower) degree of recovery uncertainty. They are found to be consistent with a number of empirical studies (e.g. Altman and Arman (2002) and Acharya, Bharath and Srinivasan (2003)) which suggest standard deviation of LGD is smaller for subordinated notes than for senior unsecured bonds. These results are also confirmed in our sub-segment analysis conducted in section VI.

Table 11: Discount Rate in Percent: Instrument Type

	Most Likely Estimate	Standard Deviation	90% Confidence Interval	
			Lower	Upper
Bank Debt	**13.3**	6.7	2.3	24.3
Senior Secured Bonds	**11.0**	6.9	-0.3	22.2
Senior Unsecured Bonds	**27.5**	3.1	22.4	32.7
Senior Subordinated Bonds	**3.8**	5.7	-5.6	13.2
Subordinated Bonds	**8.9**	3.8	2.7	15.1

Risk-Return Trade-Off

To complete this section, we would like to examine if there is an appropriate risk-return trade-off in the investment of defaulted instruments. Does a defaulted instrument of a higher LGD risk generate a higher return than one of lower LGD risk? We use the minimum sum of the squares of errors (SSE) ε_i of realized recoveries (normalized by the square root of time-to-recovery) to proxy for LGD risk for each segment. If there is a certain degree of risk-return tradeoff, we expect to find a positive relation between the discount rate and SSE.

It is indeed the case when we regress the estimated discount rate against an intercept and SSE across all segments (with more than 10 observations) examined in this section. The results are reported in equation (9). The R-square is found to be 11% and the slope coefficient of 0.123 is highly statistically significant with a t-statistic of 12.4.

$$\hat{d} = -0.01205 + 0.123 * SEE \qquad (9)$$

VI – Validation of Results: Sub-Segment Analysis

In this section, we examine the robustness of the differences in discount rates across the segments by controlling for other ways to segment the data. For example, we would like to answer the following questions: Is the discount rate during an industry-specific stress period still statistically significantly higher than that during a non-stress period if we only consider senior unsecured bonds? Is the discount rate of those instruments with some DA and some DC still statistically significantly higher than those with no DA and no DC if we only consider facilities issued by the non-technology sector?[24]

In order to answer these questions, we repeat the analysis at the sub-segment level crossing all the segments we considered previously. For example, using the proposed methodology, we solve for the point estimate and the (asymptotic) confidence interval of the discount rate of the sub-segment consisting of instruments issued by the non-technology sector and at the same time with some DA and some DC. The point estimate is found to be equal to 33.3% and the corresponding 95% confidence interval is from 26.0% to 40.5%. We can then check if the estimated discount rate of this sub-segment is statistically different from the corresponding estimated discount rates at the segment-level. Specifically, we want to check if respectively the discount rate obtained for all instruments in the non-technology sector and the discount rate obtained for all instruments with some DA and some DC actually lie within the above interval. As reported in the previous section, the former is 13.0%, while the latter is 29.3%. Since the latter lies within the above confidence interval of the sub-segment discount rate estimate, we cannot conclude that the non-technology sub-segment within the segment of some DA and some DC behaves differently from other sub-segments. However, since the former lies outside the confidence interval, we can conclude that the sub-segment of some DA and some DC is likely to have higher discount rate within the non-technology sector.

We conduct the above analysis for each of the sub-segments and the results are reported in Table 12. We only present the estimated discount rates of those sub-segments which are statistically (at 95% confidence level) significantly different from that of the corresponding segment (those bold figures presented along the diagonal of Table 12).[25] Those discount rates that are significantly higher (lower) than the corresponding segment discount rate are presented in blue (red) and in brackets (square brackets). We should interpret the discount rates of the sub-segments presented along each row and compare them with the value at the diagonal (i.e. the segment discount rate) when we look for statistically significant differences. Let us again use the non-technology sector as an example. The segment discount rate is 13.0% (the value along the "non-technology" row of Table 12 which is at the diagonal). Within this segment, the sub-segment of "No DA and No DC" has an estimated discount rate of -7.0% which is significantly lower than 13%. On the other hand, the sub-segments of "In Industry-Specific Stress Period", "Some DA and Some DC" and "Senior Unsecured Bonds" have discount rates of 21.0%, 33.3% and 26.2% respectively which are all significantly higher than 13.0%. Since

[24] The reader will find out later that the answers are "yes" for both of these questions.

[25] To make sure the results will not be distorted by a few outliers, we only consider those sub-segments with more than or equal to fifty LGD observations.

discount rates are not reported for all the other sub-segments, they are not found to be statistically different from 13.0%. The other rows can therefore be interpreted in the same fashion.

We can examine the robustness of the results presented in the previous section by considering the results reported in Table 12. Except for within the segment "No DA, Some DC", estimated discount rates of secured and unsecured instruments are not significantly different from the corresponding segment discount rate.[26] Any difference between the secured and unsecured instruments in terms of discount rate is therefore not as strong as those obtained by segmenting the instruments in other ways (discussed subsequently).

The difference in discount rates between IG and NIG is found to be robust after controlling for a number of other variables. Within five segments (namely "Unsecured", "Technology", "In Market-Wide Stress Period", "In Industry-Specific Stress Period" and "Senior Unsecured Bonds"), the NIG sub-segments have significantly lower discount rates than the corresponding segment discount rates. At the same time, the IG sub-segment within the "No DC, Some DA" segment has estimated discount rate which is significantly higher than the segment value.

From Table 12, the robustness of the previous findings of the discount rate for the technology sector being higher than that of the non-technology sector and the discount rate for instruments defaulted during the market-wide stress periods being higher than during the non-stress periods are however questionable. For example, if we only consider senior unsecured bonds, the discount rate of those defaulted during the market-wide stress (non-stress) periods is actually significantly lower (higher) than the segment discount rate of 27.5%.

From Table 12, the difference between industry-specific stress and non-stress periods is found to be very robust. For all the cases which are statistically significant, instruments defaulted during the industry-specific stress (non-stress) periods consistently have significantly higher (lower) discount rate after controlling for other variables.

The findings in the previous section regarding DA, DC and instrument type are also confirmed by the results reported in Table 12. For example, "No DA, Some DC" has significantly higher discount rate even one confines herself to consider only unsecured instruments, the non-technology sector or during periods other than the market-wide stress periods. On the other hand, the discount rates estimated for the "No DA and No DC" sub-segments are consistently lower than the corresponding segment discount rates. In terms of instrument type, senior unsecured bonds always have higher discount rate, while senior subordinate bonds lower discount rate than the segment value.

The results reported in Table 12 also suggest that instrument type by itself cannot fully characterize the recovery risk (and thus the discount rate) of the instrument. Controlling

[26] Within the "No DA, Some DC" segment, the discount rate of unsecured instruments sub-segment is estimated to be equal to 31.8%, which is significantly different from the segment discount rate of 21.2%.

for instrument type, one can still observe a large variation of discount rates across different DA and DC. For example, within the segment of senior unsecured bonds, there is significantly large variation in discount rates across different sub-segments. Specifically, those with some DA and some DC have significantly higher discount rate (70.3%), while those with no DA and no DC have significantly lower discount rate (1.6%).

To summarize, the analysis conducted in this section confirm a number of our findings in the previous section. Specifically, (1) whether the instrument is IG or NIG; (2) whether it defaults during the industry-specific stress periods or not; (3) the existence of DA and/or DC; and (4) its instrument type are found to be important determinants of recovery risks. However, the results do not strongly support the Secured vs. Unsecured differentiation, which is likely due to data constraints.[27] The impact of industry characteristics (e.g. technology vs. non-technology) and market-wide factors (e.g. stress vs. non-stress) is found to be inconclusive.

[27]The data does not allow us to distinguish the level of security, only the security type. That is, we could have two loans listed as secured, but the first one may be 100% secured, whereas the second one may only be 5% secured. It is very possible that this limitation weakens the significance of the Secured vs. Unsecured differentiation.

Table 12: Estimated Discount Rates of Sub-Segments

	Secured	Unsecured	IG (earliest)	NIG (earliest)	Technology	Non-Technology	**Mkt. stress period** in stress	not in stress	**Industry stress period** in stress	not in stress	**DA & DC** No DA, some DC	No DA and DC	No DC, some DA	Some DA and DC	**Instrument type** Bank debt	Senior secured bonds	Senior unsecured bonds	Senior sub bonds	Sub bonds
Secured	**11.8**																		
Unsecured		**14.3**		[4.4]					(22.0)		(31.8)	[-6.8]		(32.1)			(27.5)	[3.8]	
IG (earliest)			**22.8**																
NIG (earliest)				**6.4**															
Technology				[-2.4]	**24.4**		[-28.0]				(44.5)			[-49.0]					
Non-Technology						**13.0**			(21.0)			[-7.0]		(33.3)			(26.2)		
Mkt. stress period																			
in stress					[-28.0]		**15.7**												
not in stress				[3.8]	(37.0)			**13.6**	(24.7)	[3.9]	(24.9)	[-5.1]	[4.7]	(30.5)			(36.1)	[-9.8]	
Industry stress period																			
in stress				[9.6]					**21.5**				[11.0]	(35.9)			(46.6)	[4.5]	[5.6]
not in stress							(21.1)			**8.1**									
DA & DC																			
No DA, some DC		(31.8)			(44.5)		[-2.2]				**21.2**						(33.0)		
No DA and DC												**0.9**							
No DC, some DA			(23.8)										**8.6**					[-16.4]	
Some DA and DC					[-49.0]									**29.3**			(70.3)		
Instrument type																			
Bank debt															**13.3**				
Senior secured bonds																**11.0**			
Senior unsecured bonds				[2.2]			[11.8]	(36.1)	(46.6)	[8.8]		[1.6]	[16.6]	(70.3)			**27.5**		
Senior sub bonds							(28.5)						[-16.4]	(19.5)				**3.8**	
Sub bonds																			**8.9**

Note: Those estimated sub-segment discount rates that are statistically significantly higher (lower) than the corresponding segment discount rate (at the diagonal) are presented in blue (red) and in brackets (square brackets).

VII- Regression Analysis

In section VI, we check for the robustness of our results obtained in section V by considering the interactions of different ways to segment the dataset in a *two-dimension* analysis. For example, we investigate the interaction between industry-specific stress period and instrument type, and conclude that within the segment of Senior Unsecured Bonds, the estimated discount rate during the industry-specific stress periods is still significantly higher than average. In this section, we would like to take one step further by conducting a multi-dimensional investigation through regression analysis. We want to study if the variation of the returns on defaulted instruments can be explained by the various characteristics of the instruments if we consider all the factors at the same time. The dependent variable is the annualized internal rate of return on an individual defaulted instrument, assuming the instrument is bought 30 days after default and is held until it emerges from bankruptcy.[29]

The explanatory variables considered here are those found out to be important determinants of recovery risk in section V and VI, namely (1) whether the instrument is secured or not; (2) whether the earliest S&P's rating is investment grade or not (IG vs. NIG); (3) whether the default event occurs during an industry-specific stress period; (4) whether there is "No DA and No DC" or "Some DA and Some DC"; (5) whether it is Senior Unsecured Bonds or Senior Subordinated Bonds (i.e. its instrument type).[31] In the regression analysis, we also control for the variation of trading prices (30 days after default) and the weighted average time-to-recoveries of the defaulted instruments.[32]

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[29] The internal rate of return on an individual defaulted instrument is not the expected return on the instrument, which we discuss in the previous sections. The former is the realized return and can be substantially different from the expected return due to recovery uncertainty. A negative realized return, therefore, does not suggest the investors price the instrument using a negative discount rate. In order to ensure our results will not be affected by any potential outliers, we ignore those realized returns which are either higher than 500% or lower than -75% in the subsequent regression analysis.

[31] We ignore the industry effect (i.e. technology vs. non-technology) and market-wide stress situation, of which their relations with estimated discount rates are found to be weak or inconclusive in section V and VI.

[32] Because there can be multiple recoveries paid out at different times during the recovery process, we use average time-to-recovery weighted by the nominal recovery values. Our analysis suggests that using the time-to-recovery of the final cash flow rather than the weighted average time-to-recovery does not affect the conclusions drawn in this study.

The most general version of the regression equation we consider is:[33]

$$\begin{aligned} r_i &= c + a_1 P_i + a_2 Sec_i + a_3 IG_i + a_4 IndS_i + a_5 DADC_{1,i} + a_6 DADC_{2,i} \\ &\quad + a_7 Ty_{1,i} + a_8 Ty_{2,i} + + a_9 TTR_i + \varepsilon_i \end{aligned} \tag{10}$$

where

r_i = internal rate of return on default instrument *i*;
P_i = trading price of instrument *i* 30 days after default (in $ per $1 nominal value);
Sec_i = dummy variable: equal to "1" if instrument *i* is secured, "0" otherwise;
IG_i = dummy variable: equal to "1" if instrument *i*'s earliest S&P's rating belongs to investment grade, "0" otherwise;
$IndS_i$ = dummy variable: equal to "1" if instrument *i* defaults during industry-specific stress period, "0" otherwise;
$DADC_{1,i}$ = dummy variable: equal to "1" if there is no DA and no DC, "0" otherwise;
$DADC_{2,i}$ = dummy variable: equal to "1" if there is some DA and some DC, "0" otherwise;
$Ty_{1,i}$ = dummy variable: equal to "1" if instrument *i* is Senior Unsecured Bond, "0" otherwise;
$Ty_{2,i}$ = dummy variable: equal to "1" if instrument *i* is Senior Subordinated Bond, "0" otherwise;
TTR_i = the weighted average time-to-recovery of instrument *i* (in years).

The regression results are reported in Table 13. We present the coefficients together with the corresponding *t*-statistics (in italic).

[33] In this study, we consider various single-explanatory variable versions of equation (10) together with the general version of the multiple regression.

Table 13: Regression Analysis: Coefficients together with the corresponding *t*-statistics (in italic)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	**0.428*** **	**0.417*** **	**0.335*** **	**0.462*** **	**0.426*** **	**0.591*** **	**0.412*** **
	9.264	*10.386*	*6.290*	*10.899*	*9.083*	*6.641*	*5.020*
Trading price (per $1 nominal)						**-0.484*** **	
						-4.890	
Secured	-0.015					0.104	-0.062
	-0.293					*1.274*	*-0.819*
IG (earliest S&P rating)		**0.187** **				**0.264*** **	**0.182** **
		2.210				*2.956*	*2.052*
Industry-Specific Stress Period			**0.120** **			**0.085***	**0.144*** **
			2.454			*1.684*	*2.902*
DA and DC							
No DA and No DC				**-0.249*** **		**-0.231*** **	**-0.265*** **
				-3.534		*-3.251*	*-3.708*
Some DA and Some DC				-0.056		-0.033	-0.022
				-0.837		*-0.475*	*-0.312*
Instrument Type							
Sen. Unsecured Bonds					0.033	0.033	-0.020
					0.620	*0.437*	*-0.261*
Sen. Sub. Bonds					-0.088	-0.135	**-0.144***
					-1.353	*-1.608*	*-1.695*
Weighted ave. time-to-recovery (year)	**-0.103*** **	**-0.110*** **	**-0.093*** **	**-0.102*** **	**-0.103*** **	**-0.116*** **	**-0.103*** **
	-4.902	*-5.241*	*-4.414*	*-4.956*	*-4.958*	*-5.407*	*-4.753*
R-square (adjusted)	0.025	0.030	0.031	0.036	0.027	0.071	0.048
p-value of F test	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: - Coefficients significant at 10%, 5% and 1% level are marked with *, ** and *** respectively
- p-value of F test is for the hypothesis test where all coefficients are zero

Consistent with the findings in the previous sections, the single-variable regression results (regression equations (1) to (5) in Table 13) suggest the return on a defaulted instrument is statistically significantly higher if its earliest S&P rating is IG, if it defaults during the industry-specific stress period and if it has no DA and DC. The signs of coefficients of "Secured" and "Instrument Type" are consistent with the results documented in section V and VI, however they are not statistically significant. Moreover, there is a negative relation between realized return and time-to-recovery.

The multiple regressions are reported as regression equations (6) and (7) in Table 13. The coefficients of "IG", "Industry-Specific Stress Period" and "No DA and No DC" remain statistically significant and of the expected sign. In addition, the negative coefficient for "Senior Subordinated Bonds" becomes statistically significant at 10% level (in regression equation (7) in Table 13). It therefore confirms our previous conclusion that senior subordinated bonds have significantly lower estimated discount rate. As expected, the coefficient of "Secured" is negative, however it is not found to be statistically significant. Finally, the realized return is found to be negatively related to the trading price (see regression equation (6) in Table 13). It is as expected given the limited upside gain of fix-income instruments.

VIII- Conclusion

In this study, we empirically solve for the discount rate required to estimate economic LGD from workout recoveries. We find out that discount rates vary by security, initial obligor's rating, whether or not the industry is in stress condition at the time of default, relative seniority to other debt and instrument type. The sub-segment level analysis and the multiple regression analysis conducted on the internal rates of returns of defaulted instruments confirm the statistical significance of the results and the robustness of our conclusions.

Discount rates for those instruments which default during the industry-specific stress periods are double those which default during other periods. During the market-wide stress periods when default rates are, in general, higher, not only expected recoveries are lower, but also the discount rate tends to be larger, which results in an even larger economic LGD and more pronounced PD and LGD correlations. We provide a framework for banks to determine the appropriate discount rate to be used in their LGD estimation process in order to fulfill the requirement under the advanced IRB approach of Basel II regulatory capital calculation and their internal economic capital measurement.

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